Gilles Deleuze

Seminar on Leibniz and the Baroque – Principles and Freedom

Lecture 08, 27 January 1987: Principles and Freedom (3) -- Tales of the Compossible and the Incompossible; Invited Presentation on Neighborhoods and Singularities

Initial transcription by WebDeleuze; Augmented transcription and translation by Charles J. Stivale[1]

Part 1

We will have to work all the harder today since our session is short because I have some meetings that are indispensable for your future. The meetings start around noon. So this will be a shorter session.

So here's where we are. The first thing I would like… We find ourselves faced with three questions, three questions to specify. So we need to specify these so that I am happy. These three questions will serve as our conclusion.

The first question, we saw it the last time, is the notion and the extreme importance of the notion of singularity, and I think that singularity or singular point is a notion of mathematical origin and that appeared with the beginnings of the theory of functions. Historians of mathematics correctly consider that the theory of functions is, no doubt, the first great formulation on which what we call modern mathematics depends, the theory of analytical functions. And Leibniz is at the base of this theory of functions. Leibniz's importance in mathematics is without doubt since, in his mathematical works, he elaborates a theory of function to which there will not be, I don't say anything more to be developed, but in which there will be very little to change. So it's a fundamental mathematical act that orients mathematics towards a theory of functions.[2]

And the singular points, or singularities, are the essential instrument of this theory; only Leibniz is not satisfied with being the first great mathematician to develop an entire theory of functions. I am not saying that he invents it since it's in the seventeenth century that the rudiments of a great theory of functions is sketched out. But not only is Leibniz that [a great mathematician], the concept of singularity will be unleashed and becomes in his works a philosophical-mathematical concept, and in what sense? In the exact sense in which – generally – we can say: singularities – you would expect a lot of them; we have seen that there are several sorts -- and this will be a topic for us, to sort out singularities, in the Leibnizian sense of the term singularity.

And in the first sense of the term singularity, what is a singularity for Leibniz? I would say very summarily that a singularity is an inflection, or if you prefer, a point of inflection, and the world is an infinite series of inflections. The world is the infinite series of possible inflections. All of this, we have seen. So my first question-conclusion is: what is a singularity, or what is a singular point, once it's been said that – generally – we can say that a singularity is an inflection, or that a singularity is there where something happens in a curve? Thus, from the beginning, our idea of the surface with variable curvature that is the fundamental theme that seemed to us to be

Leibniz's, is inseparable from a technique and from a philosophy of singularities and of singular points.

I don't need to insist, I think, on the novelty of the sense of such a notion, since certainly earlier, logic was familiar with the universal, the general, the particular, the singular. But singularity in the sense of a singular point or what happens has a line, that is something completely new, and in fact, it's of mathematical origin. Starting at this level, I can then define an event philosophically as an aggregate of singularities. I would say at that moment that the notion is not even only of mathematical origin, but of physical origin. A critical point in physics, evaporization, crystallization, whatever you like, a critical point in physics presents itself as a singularity. All that, you grasp, is already an aggregate of problems, the arrival of this mathematical-physical-philosophical notion, the singular point, so let us give praise to Leibniz.

There you have a first group of questions that, for us, are well brought forth, but you understand that this is material under development and research.

Second question, or second foreshadowing (*pressentiment*) that we have: perhaps that, between two singularities, there is an entirely original type of rapport, and a logic of the event requires that this type of rapport be specified. What is a rapport, and what is the type of rapport between singularities? And the last time, I proposed a hypothesis starting from the following idea, a notion as bizarre as the one that Leibniz introduces in telling us: if you take an aggregate of possibles, they are not necessarily compossibles, so the relation of compossibility and incompossibility would be this type of relation between singularities. "Adam non-sinner" is incompossible with the world in which Adam sinned. Once again, this is what matters, understand well: "Adam non-sinner" is contradictory to "Adam sinner", but it is not contradictory to the world in which Adam sinned. Simply, between the world in which Adam sinned and the world in which Adam does not sin, there is incompossibility.

So God's situation when it created the world is very strange, you see, and that belongs to some of the most famous ideas of Leibniz. God's situation when it created the world is that God finds itself in the situation in which it chooses between an infinity of possible worlds. It chooses between an infinity of equally possible worlds, but that are not compossible with one another. In God's understanding, there is an infinity of possible worlds, and God is going to choose. Among the possible worlds, which are not compossible with each other, it is going to choose one among them.

Which one? Fortunately we don't yet have to deal with this question, but it's easy to guess Leibniz's response: it [God] is going to choose the best, the best. It is going to choose the best of possible worlds. It can't choose them all at once, they are incompossible. So it is going to choose the best of possible worlds, a very, very curious idea, but what does the best mean, and how does it choose the best? There has to be some kind of calculus! What will the best of possible worlds be, and how does he choose one? Isn't Leibniz going to enlist in a long theory of philosophers for whom the superior activity is the game? Only to say that, for many philosophers, the superior or divine activity is the game, which isn't saying much, because it's a question of knowing which game we are talking about. And everything changes according to the nature of the game. It is well known that Heraclitus already invoked the game of the child-player, but everything depends on what game he is playing, this child-player. Does Leibniz's God play the same game as Heraclitus's child? Will this be the same game that Nietzsche invokes? Is it yet again the same game as Mallarmé's?

Perhaps we'll have to, and Leibniz will force us to create a theory of games, even not creating a theory of games, something that he found exciting. In the seventeenth century, the great theories of games begin. Leibniz will lend himself to this effort, and I add the following erudite comment: it's that Leibniz knows the game of "go", that's really interesting [*Laughter*], he knows go, and in a quite astonishing little text, he makes a parallel between go and chess, and he says that, in the end, there are two kinds of games. He doesn't call it "go"; he says "a Chinese game," and he says that the great difference between go and chess – and he says something quite correct – chess belongs to games in which it's a matter of taking. One takes pieces -- You see the classification of games outlined here -- one doesn't take pieces in the same way as in chess and checkers, since there are several modes of capture; but these are games of capture. Whereas in go, it's a question of isolating, of neutralizing, of surrounding, not at all taking, [but] of inactivating.

So I add the "scholarly remark", that in the editions of Leibniz of the nineteenth century, the game of go is so little known that, in reference to this text by Leibniz, there is a note, for example in the Couturat edition, at the start of the twentieth century, Couturat who is a very good specialist both in mathematics and in Leibniz, there is a note by Couturat about Leibniz's allusion to this Chinese game. He says that this refers to… He describes a little and says "according to what a China specialist has told us." So, it's very curious since according to Couturat's note, go was not at all known at that time. It's importation to France is quite recent. Anyway, in short, I'm wasting time. [*Laughter*] This was just to tell you … to tell you what? Oh right, as a result of what calculus, of what game, did God choose a determinable world as the best. Ok, we will leave that aside because, you understand, it's not difficult, the answers aren't difficult, and for the moment, we are swimming in what's difficult.

What I'm saying is, what matters to us, and this is my second question, is this: what is the type of relation that allows us to define compossibility and incompossibility? At our last meeting, I was rather forced to say that Leibniz's texts were somewhat lacking on this topic, but that we had the right to attempt an hypothesis, and the hypothesis that we attempted was the following: couldn't one say that there is compossibility between two singularities when the prolongation (*prolongement*) of one into the neighborhood of another gives rise to a convergent series, or on the contrary, for incompssibility, when the series diverge? It's therefore the convergence and divergence of series that would allow me to define the relation between compossibility and incompossibility. So compossibility and incompossibility would be the direct consequences of the theory of singularities. This is my second problem, and I insisted on this: these are problems. This is the second problem that we were able to derive from our previous meeting.

Third, and final, problem, is that, henceforth, I had at least – a considerable advantage, but we'll see – I had at least a final hypothesis about this fundamental question for Leibniz: what is individuality or individuation? Why is this a fundamental question for Leibniz? We have already seen it, if it is true that any substance is individual, if it is true that the substance is the individual notion designated by a proper name, you, me, Caesar, Adam, etc. The question "what does individuation consist of ?", "what individuates substance if all substance is individual?", becomes fundamental. My answer or my hypothesis was this: can't we say that the individual, the individual substance, is a condensation, a condensing (*condensé*) of compossible singularities, that is, convergent? In the end, this would be a definition of the individual; there is nothing more difficult to define than the individual. So we would have done… If we can say this, I would then say, almost, that individuals are singularities of a second type.

What would that mean, a condensing of singularities? For example, I defined Adam the individual through a primary singularity, and I return to the text of the Letters to Arnauld: "first man"; second singularity, "in the garden"; third singularity : "having a woman born from his own rib" ; fourth singularity : "having endured a temptation". You see all kinds of [*First gap in WebDeleuze transcript*] problems is it the case that in order to define an individual, an infinite number of singularities is required or not ? There we have a problem. Another problem: I can only define the individual as a condensing of singularities if the singularity does not already implicate the individual. About this point, I'm greatly interested. In fact, the singularity does not already implicate the individual. The individual is what? It's the subject that envelops singularities; it's the subject that includes singularities. As we've seen, it's the soul. [*Pause*] But [*End of first gap in WebDeleuze transcript*] singularities pre-exist the subject, in what sense? A perfect expression exists for us; we will say of singularities that they are pre-individual.

Henceforth, there is no vicious circle, which would be quite vexing, of defining the individual as a condensing of singularities if singularities are pre-individual. "Condensing" (*condensé*) means what? All sorts of texts by Leibniz tell us and remind us that points have the possibility of coinciding, and it's even for that reason that points are not constitutive parts of extension. If I have an infinite number of triangles, for example, or of angles, if I have an infinite number of angles, I can cause their vertices to coincide. I would say that "condensing of singularities" means that the singular points coincide. The individual is a point, as Leibniz says, but a metaphysical point; the metaphysical point is the coincidence of an aggregate of singular points. Hence the importance – but this is what we have done since the beginning, but I insist on justifying it perpetually --, it is well understood that Leibniz repeats to us constantly: there are only individual substances. In the end, the only thing real, understand the only thing real are individual substances. [*Pause*]

But that does not prevent… we have seen, and it's what we have done, we had to begin from the world, that is: we had to begin from inflection. We had to begin from an infinite series of inflections. It's only secondarily that we noticed that inflections, and the world itself, exist only in individual substances that express it [world]. [*Pause*] But that does not prevent individual substances from resulting from the world, which is what I told you. We had to maintain absolutely the two propositions at once: individual substances are for the world, and the world is in the individual substances. Or, as Leibniz says: God did not create "Adam sinner" – that's the key text for me since, without this text, everything we have done, the order that we followed in the first trimester, that is, going from the world to the individual substance, would not be valid. -- God did not create "Adam sinner"; it created the world in which Adam sinned, once it's been said that the world in which Adam sinned exists only in individual notions that express it, the notion of Adam and the notions of all of us who live under original sin. Good.

So you see… My third point is this whole sphere of the problem of individuation in which I believe Leibniz, there as well, is the first. If I sum up the three points, I am saying that, among all the fundamental things that Leibniz brings to philosophy, there is first off the eruption of the mathematical-physical-philosophical notion of singularity, to which my problem responds, "but, in the long run, what is a singularity?" because we will never finish with the singularity as constitutive element of events. A logic of the event, a mathematics of the event, it's a theory of singularities. And, in mathematics, that overlaps with the theory of functions, but we call not only for a theory of functions, but also for a logic of the event.

Second point: the types of relations from one singularity to another, compossibility, incompossibility, convergent series, divergent series, and what are the consequences of all that for the understanding of God, and for the creation of the world, and for the game of God, if God creates, that is, chooses the best of worlds through a kind of calculus or game? Third point: what is individuality if we start from the idea that it condenses a certain number of singularities, or an infinity of singularities, etc., these singularities being, henceforth, necessarily pre-individual?

That makes three tough problems. I would just like – before… here this is quite simple, before calling on those people who are more competent than me – I'd like to draw from this some restful consequences. You see this really curious situation, the compossible, the incompossible. In the understanding of God, an infinity of possible worlds is agitated. There Leibniz plunges in deeply. I apologize to those who were here two years ago; I already spoke about that regarding another matter, regarding the true and the false, and yet it seems that evidently I have to address it again, but I am going to go rather quickly. I am speaking for those who were not here. There are three fundamental texts that you must consider.[3]

There are three fundamental texts that you must consider. The first is quite famous, by Leibniz himself, the *Theodicy*. In the *Theodicy*, part 3, paragraphs 413 and after, it's an eminently Baroque text, to return to our theme. What does one call a Baroque tale? For example, Gérard Genette and other critics considered this, and summarily, they agree in telling us this: at first glance, what characterizes Baroque texts is above all the nesting (*emboîtement*) of tales one within another, on one hand, and on the other hand, the variation of the relation of narrator and narration, both becoming but one. In each tale nesting into another corresponds, in fact, to a new type of narrator/narration rapport.

If you take, starting from paragraph 413, the very curious tale that Leibniz tells, and which is extremely beautiful – in the *Theodicy* – you will see that it's a typically Baroque story since it begins from a dialogue between a Renaissance philosopher named [Laurentius] Valla…[4] , [*Start of transcript gap 2 at WebDeleuze*] a dialogue between Valla and Antoine [Antonio Glarea] on the theme, "Is God responsible for evil?" And in this dialogue [*End of gap 2 WebDeleuze*] a Roman character is evoked, Sextus, the last king of Rome who exhibited evil passions and, notably, raped Lucretia. Some say that it's his father who raped Lucretia, ok, but in the tradition to which Leibniz refers, it's Sextus who raped Lucretia. And the question is: is this God's fault? Is God responsible for evil?

To this first tale, the dialogue between Valla and Antoine, this first tale nests into a second tale which is Sextus going to consult Apollo, to tell him, but really, Apollo, what is going to happen to me? Then a third tale is juxtaposed to this: Sextus is not satisfied with what Apollo tells him, and he seeks out Jupiter himself. He addresses himself directly to Jupiter to have a first-hand answer. [There are] variations of the tale. There, in the Sextus-Jupiter discussion, there is a new character named Theodorus, the High Priest, loved by Jupiter. And [in] a new tale, it's Theodorus, observing the dialogue between Sextus and Jupiter, who says to Jupiter: but still, you didn't answer him very well, to which Jupiter says: Go see my daughter, Pallas. So the last tale nests into the others: Theodorus goes to see Pallas, Jupiter's daughter. You see that all this creates quite a nest of overlaps. And then! [*Deleuze breaks out laughing*], Theodorus falls asleep! [*Laughter*] This is typically Baroque. Baroque novels are just like that. So I cannot believe that Leibniz… He knows perfectly well what he is doing; in this ending of the *Theodicy* which is entirely crazy, he knows perfectly well what he is doing. It's a grand Baroque imitation and, once again, he knows it.

So Theodorus falls asleep, but he dreams. He dreams that he speaks to Pallas, and there Pallas tells him: come follow me! It's not over. She leads him to see a splendid transparent pyramid. This is Theodorus's dream. It's the palace of the fates, on which I stand guard, Pallas tells him. She says that Jupiter comes sometimes to visit these sites for the enjoyment of reviewing things and to renew his own choices. God comes to visit this architecture, this transparent architecture. What is this transparent architecture? It's an immense pyramid, which indeed has a vertex, but that has no end.

You sense immediately that something is coming. This means that, in the infinity of possible worlds, there is indeed a world that is the best, but there aren't any that are the worst. On the side of the depths, it extends to infinity, but not on the side of the heights. There is a maximum, but there is no minimum. That interests us because one must consider everything mathematically. In the lists of everything that is a singular point, we will see that there is a moment in which arises – not at all for the moment – the idea that there are maxima and minima. I believe the maxima and the minima are not of the same kind in Leibniz. On the level of worlds, there is indeed a world that is the best, but there is no world that would be the worst. There's a maximum; there is no minimum.

So I have my endless pyramid with its vertex, and way up at the top… but notice that this poses a problem; the text is splendid, I hope you'll read it, but that poses a problem because how do we organize it, even if I attempt to draw an illustration? I have my pyramid. Way up at the top there is an apartment – "apartment" is the word that Leibniz uses. You recall our stories, the upper floor, the lower floor, all that. You will see all that returning in this admirable text. -- There is an apartment that culminates at in points, if I understand well; it occupies the whole upper region of the pyramid. And in this apartment, a Sextus lives. Fine. Below, Leibniz tells us, there are other apartments, and here it gets complicated. I consider all these apartments, and it's not easy; how are they organized? In my view, it's not possible that there are any with the top below; in other words, grasp this: how to fill a pyramid and with what figures? I would say, what is the figure of the apartments? It's a problem that mathematicians know well and that's an exciting problem.

On the simplest level, given a surface… [*Interruption of the BNF & YouTube recording; text continues that to the transcript from WebDeleuze*] [36:14]

Part 2

… how does one divide it in such a way that there is no empty part? More simply, how does one pave a space? The problems of paving are also problems of architecture, but also problems of mathematics. For example, can you pave a circle with circles, or will there be empty parts? Given a surface, with what can you pave it? The tradecraft of a paver seems like nothing big, but it's one of the most beautiful trades in the world, see? It's a divine activity, paving. The proof is that [*Continuation of the BNF & YouTube recording*] Leibniz, in a famous text titled *On the radical origin of things* – he had a genius for creating titles; what is lovelier than a book titled *On the radical origin of things*, especially as this book is 15 pages long -- [*Laughter*] and indeed Leibniz explicitly refers to it, paving, regarding the creation of the world by God. That is, he assumes – and this is something he does not believe, but it matters little – he assumes that space is assimilable to a given surface, and he says: God necessarily chooses the world that fills this space the best and to the maximum. In other words, God chooses the world that best paves the space of creation.

So how am I going to pave my pyramid of apartments in such a way that there is no empty space? It's interesting. One must assume that, if these are little pyramids, no apartment has its point downward, otherwise that doesn't work at all. You see, it's in order to open you to immense problems that I tell you all this.

But then in the lower apartments… each apartment, Leibniz tells us, I'm not sure where, but believe me, each apartment is a world. [*Deleuze looks through the text for the quote*] No, hmm… Ah, hé, hé, I've located the text: "Thereupon the Goddess led Theodorus into one of the halls of the palace: when he was within, it was no longer a hall, it was a world" [*Theodicy*]. I have the impression that it's the entryway in Baroque style. You enter into the Baroque room and, at the same time that you enter, it's no long a room, it's a world. You have a first apartment in which you have a Sextus, and then you have another apartment, below, there is not a floor sufficiently low, there are always lower floors, but there is a floor that is the highest. So, on the upper floor, you have a Sextus, in the following floors, you have other Sextuses. Consider the problem: why are these Sextuses? That's going to be a problem for us.

So that's where it gets complicated, but everything is important in this text which is so delightful (*gai*) ; he says : each of the Sextuses, in the apartments, has a number on his forehead, a number 3000, 10000, some as it's infinite from the base, you have a Sextus that has the number 1,000,000. The Sextus in the apartment up above has number 1. Why does he have a number? It's because at the same time – you recall the text I read to you[5] -- the upper room was a reading chamber, in the Baroque style. In each apartment there is a great volume of writings. [*Deleuze reads with great feeling*] "Theodorus couldn't keep from wondering what that meant? Why is there a great volume of writings? It's the history of this world, Pallas answers. It's the history of this world that we are visiting right now, the goddess tells him. This is the book of its fates. You have a number on the forehead of Sextus, look at the spot in this book that it marks. Theodorus looks for it and finds the story of Sextus, the entire story. However, I already saw Sextus in his transparent apartment," yes indeed! Yes, I saw him, and he was imitating a sequence; for example, he was raping Lucretia, or something more acceptable, he was getting crowned king of Rome. I was noticing that; theater, theater. But it does not include everything. In other words, the entirety of the world to which that Sextus belongs, that is, the entirety of the world with which that Sextus, the one who raped Lucretia and was crowned the king of Rome, with which this Sextus is compossible, I didn't see him, I read it in the book. You see the combination reading-seeing proper to the Baroque, there as well, what we called the last time the emblem, in saying that the Baroque is emblematic; we find it again here completely.

Let's return [to the text]. I am wandering. So [there is] the Sextus up on top, good. But below, I see a Sextus who goes to Rome, but renounced being crowned. As Leibniz says, he buys himself a little garden and becomes a rich and respected man. It's another Sextus, he has a different number on his forehead. I would say: This Sextus number two is incompossible with the apartment on top, with the world above, with the world 1. And then I see a third Sextus, who renounces going to Rome, and goes somewhere else, to Thrace, and he gets crowned king of Thrace. He doesn't rape Lucretia. Let's suppose etc. … etc. … to infinity. You see all these worlds are possible, but they are incompossible between themselves.

And what does that mean? That means that there is divergence, there is a moment in which it diverges. Why are they all Sextuses? We return to the problem because it's very important, but one can assume that it's because a small number of singularities are common to them. All are the sons of Tarquinius, and successors to the king of Rome; but in one case, he does in fact succeed his father, in another case he renounces the succession and leaves Rome, in another case he renounces the succession but stays in Rome. You see that the divergences do not pass from one world to another. The divergences that define incompossibility do not necessarily pass into the same spot. That's what is very important: I have a network of divergences that do not begin in the same singularity, or that do not begin in the passage of the same singularity with another. You have this extremely joyful tableau of incompossible worlds. An aggregate of compossibility, an aggregate of compossible singularities defining a world, and God chooses, he chooses the best of possible worlds in all this.

So I was saying that, it's here that very quickly I just want to allude to two fundamental texts, two typically literary Leibnizian texts.[6] One poses no problems since its author is extremely knowledgeable and created a typically Leibnizian version -- it's also very curious, without… but he has no need to cite [Leibniz] -- it's by Borges, Borges, with the title "The Garden of Forking Paths." You see the compossible… [*Laughter*] What's going on?

Hidenobu Suzuki (*seated beside Deleuze*): That it's a brilliant text.

Deleuze: What?

Suzuki: Someone was saying that it's a brilliant text.

Deleuze: Here, the incompossibility has become, under Borges's pen, the bifurcation, paths that bifurcate. I'm just reading… You can refer to this; it's in a volume titled *Fictions*, "The Garden of Forking Paths," [*Pause*] and I'll read a passage. He recounts a novel written by a mysterious Chinese writer: "Usually, in all fictions, when a man is faced with alternatives, he chooses one at the expense of the others" -- notice that this exactly God's situation in Leibniz: between incompossible worlds, he chooses one and eliminates the others. -- "In the fiction of the almost unfathomable Ts'ui Pên, he chooses all of them simultaneously." Imagine a perverse Leibnizian God, who would cause to come into existence all the incompossible worlds. What would Leibniz say? Leibniz would say this is impossible! But why is this impossible? Because in that case, God would renounce his favorite principle, which is the principle of the best, choosing the best. Supposing a God who cared not all about the best, which is clearly impossible, impossible, but suppose such God, then we slide from Leibniz to Borges. "He thus *creates* various futures, various times which start others that will in their turn branch out and bifurcate. From this comes the novel's contradictions"[7] – by the tireless Ts'ui Pên – "From this comes the novel's contradictions. Fang, let us say" -- it's a character like Sextus -- "has a secret. A stranger knocks at his door. Fang makes up his mind to kill him. Naturally, there are several possible outcomes. Fang can kill the intruder, the intruder can kill Fang, both can be saved, both can die, and so on and so on. In Ts'ui Pên's work, all the possible solutions occur, each one being the point of departure for other bifurcations."

I would say that in the understanding of God, it's exactly the same thing. In the understanding of God, all possible worlds are developed. There is simply a blockage: God only causes to pass into existence one of these worlds. But in his understanding, all the bifurcations are there; this is a vision of the understanding of God that has never been seen. It's very interesting, but this is how… I just wanted to state the way in which Borges creates a pure application, an exercise of style, that comes directly from the *Theodicy*. But what interests me more is this novel that I mentioned and that I wanted you to read, and that is even more Leibnizian, literally Leibnizian. This novel comes from someone that we wouldn't expect and who reveals himself as a great philosopher, Maurice Leblanc, a great popular novelist of the nineteenth century, well known as the creator of Arsène Lupin. But besides Arsène Lupin, he wrote some admirable novels, and better than the Lupin [novels], and notably one that has been re-edited – it's marvelous! -- in the *Livre de poche* series, called: *La Vie extravagante de Balthazar* [Balthazar's Extravagant Life]. You are going to see the extent to which this is important for us that I will rapidly summarize.

This is a very convoluted novel: Balthazar is the hero, and he's a young man working as a professor of daily philosophy, and daily philosophy is a very special philosophy, very interesting, that consists of saying: nothing is extraordinary, everything is regular, ordinary. Everything that happens is ordinary; in other words, there are no singularities; that's quite important. During the novel, all sorts of frightening misfortunes befall Balthazar, and each time, he his pursued by a timid sweetheart named Coloquinte. And Coloquinte tells him: But Monsieur Balthazar, what does the daily philosophy say, because this is not banal what's happening to us? And Balthazar scolds her saying: Coloquinte, you don't understand, all that is quite ordinary as well shall soon see. And the singularities dissolve. You recall my entire theme: how do singularities develop? By extending themselves over a series of ordinaries, into the neighborhood of another singularity.

And what carries it along? Do the ordinaries depend on singularities, or do singularities depend on ordinaries? A text by Leibniz that really appeals to me, in the *New Essays*, and that I quoted the last time, would have us believe that the answer is complex since Leibniz tells us: what's remarkable (understand: the singularity) must be composed of parts that are not remarkable. What is remarkable must be composed of parts that are not remarkable, in other words, a singularity is composed of ordinaries. What does that mean? I was telling you that it's not very complicated. Take a figure like the square which has four singularities, its four vertices, anyway its four I don't know what, its four thingies where that changes direction, its four singular points. I can say A, B, C, and D; I can say that each of these singularities is an ordinary double point since the singularity B is the coincidence of an ordinary that belongs to AB, and another ordinary that belongs to BC. Fine. Should I say that everything is ordinary, even the singularity, or should I say that everything is singular, even the ordinary? Balthazar has chosen the first viewpoint and says: Everything is ordinary, even singularities.[8]

However, some strange things befall Balthazar, since it so happens that he does not know who is father is. And as it happens, he could care less; contrary to the hero of modern novels, Balthazar has no interest in knowing who his father is, [*Laughter*] but it happens that there is an inheritance problem for which he has to learn it. And Leblanc, the immortal author of this beautiful book, of this great novel, provides three singularities that define Balthazar: he has fingerprints, it's a singularity since his prints do not resemble those of anyone else. First singularity, his fingerprints. Second singularity, a tattoo that he wears on his chest made of three letters: m Maurice, t Theodore, p Paul, MTP. And a third singularity, a clairvoyant that he visited, despite himself, told him: your father has no head. [*Laughter*] So Balthazar's three singularities are: having a headless father, having his very own fingerprints, and having a tattoo as mtp. That corresponds to Adam's three singularities – being the first man, being in a garden, and having a woman born from his rib. We can start from there.

There follows a whole series of fathers who arrive. First father, the Count de Coucy-Vendôme answers the conditions well since he died by having his throat being cut, by a bandit, the head mostly cut off. Is Balthazar his son? Starting from three given singularities, are they extended out and into the neighborhood of that singularity: being the son of an assassinated Count? No doubt, yes, in one world. In one world, that's it, that works very well. But from that point, at the moment that Balthazar is going to receive the inheritance of the Count de Coucy, he gets kidnapped by a bandit who tells him:[9] "you are the son our former boss," who was called Gourneuve. It was a notorious bandit, and not only a notorious bandit, but also the one who cut off the count's head. So, this second father assassinated the first one, and thus he complicated the network because henceforth, both of them are going to belong to a compossible world, and yet they are going to be incompossible.

But we're not done, ok? This is a long calculation. But finally Gourneuve has no head; he meets the necessary condition because he lost his head by being guillotined. And he offers a supplementary advantage since, you may have noticed that mtp hasn't yet been justified, whereas Gourneuve, in the first case, he was the boss of the Mastropieds gang, which is the gang M-T-P, thus justifying the tattoo. [*Pause*] So, there where we have incompossibility, we cannot have for father both the assassin and the one assassinated. [*Laughter*] So this is another world, as it diverges. And yet, the two fathers belong to the same compossible world, but at the same time, it's incompossible.

But there's no reason for too much concern because at the same time that Balthazar is going to be integrated into the Mastropieds gang, he is kidnapped, kidnapped by an Englishman who takes him to the Far East, where there's a war, and gives him over to a leader named Revade Pacha. And Revade Pacha tells him, "you are my son; you're my son. You are Mustapha," M-T-P, Mustapha. [*Laughter*] This is a third world. What a great Baroque novel this is, the very example of the Baroque novel, with travel, everything! So… Then, shortly thereafter, Revade Pacha gets decapitated, so everything is there: he too is without head, mtp is justified, all is well.

But at the point when everything is going wrong for Balthazar, here comes a poet named Beaumesnil, and this poet saves him. He says to Balthazar, "you're my son!" So this is the fourth one. Only Balthazar had stolen something from his previous father, Revade Pacha, taking off with Revade Pacha's treasury. And so we have the new father, Beaumesnil the poet, who goes mad! That is, he loses his head! [*Laughter*] He loses his head, and he runs off after stealing the money from his son, [*Pause*] and crying out, "It's counted, weighed, divided!" You have all recognized the famous expression, "Mane Thecel Phares". Mane Thecel Phares is M-T-P, a new world that diverges. So here we have four worlds that are all possible, but at the same time, are incompossible with one another.

So at this point, everything will then be explained to us. Everything here is a swindle. This is the problem: Is the Leibnizian God a swindler? And in this, what I'd like you to understand is that, in fact, Leibniz escapes the critique, but that his God would be a swindler if it were like Borges or like Leblanc. That is, if God caused incompossible worlds to come into existence, there we would really have a fraud.

Fortunately, the Leibnizian God is moral, that is, doesn't cause these incompossible worlds to come into existence. Why? Because the fraud is the following: a hobo arrives who is the fifth father, a hobo named Vaillant-Dufour. And Vaillant-Dufour had an idea when he was quite young. It was to create a boarding home for rich young boys who were far from their parents. You see? He had a little boarding home for rich boys, four of them, four rich boys as well as his own. So, in fact, he had the count's son, the bandit's son, the rich bandit, Revade Pacha's son, and the poet's son, as well as his own son. So the boarding home consisted of five children. And then there's a flood, and only one of the sons survived. And the hobo doesn't even know which one is his son; he doesn't know if he survived. And he tells himself, well, this is quite annoying because what can he do to keep the money and keep the families paying? So he creates a document with the fingerprints of the survivor, and he sends it to the four parents saying, "Your son survived; yours is the one that survived." You see? This isn't a dumb idea! [*Pause*] So, at the same time, he doesn't know, and in the end, is the hobo the real father? Is he the fifth father that would make… and that would unify all the incompossible worlds? Well, not even, because he doesn't know. He doesn't know if he's the father. So, in his own turn, he only belongs to one of the worlds, and he is unable to say it except through fraud. And finally, in the end, he is such an alcoholic that he had also lost his head. [*Laughter*] So this is fine. Each time there are series that swerve off, divergent series, etc. It's the mystery of incompossible worlds, or as Leibniz says, the ballet of fates.

So, I come back to this. I would like everything to be much more concrete, and I return to my question, or rather my three questions, my three questions being: up to what point could we develop a mathematico-philosophical theory of singular points? Second question: up to what point can we develop the idea of original relation, that is, irreducible to any other type of relation that would unify, positively or negatively, one singularity to another, positively in the case of compossibility or the convergent series, negatively, exclusion from the point of view of divergent singularities, or of divergent series? Third question: is it possible to define the individual as a condensing of convergent singularities, and what is the consequence for the very notion of the individual or for the principle of individuation?

So it's on this point that, if you allow, [*Deleuze speaks to his mathematician colleague, Marcel Maarek*] I would like to ask you about these three questions, or about others, I'd like to ask you if you see some research directions for all of us, for… ? [*Pause*]

Maarek:[10] I had prepared a presentation based on your last class, but since then, you have bifurcated so much that…

Deleuze: Oh, no, no, no, we can certainly return to earlier points, eh?

Maarek: So I will have to bifurcate, and so I am asking you for great indulgence. Let's say that my presentation is likely to be lateral, in principle, or if it comes back to the center, that will be fine.

I'd like to go back to a notion that is essential, that refers us to another one that we have spoken about on occasion, that you [Deleuze] have spoken about at one time, which is central in my opinion. It's the notion of singularity. So I would like to say this: singularity, how might we define something of this sort? A possible definition – I approach it as did [René] Thom, but why not? – this would be something that would occur differently than in any possible neighborhood, that is, as if something different suddenly happens from anything occurring all around. I am voluntarily using vague words because, as you will see, there lays the difficulty: it's how to define the neighborhood (*voisinage*)? So, singularity refers to neighborhood, that is, refers to the relations between the singular point and everything near it, everything immediate to it. Something different has to happen, however near that it might be. This is perhaps the problem about which mathematicians are the most deeply concerned. Earlier you were giving an utterly striking example of the problematic for mathematicians at the end of the nineteenth century of the problem of singularity.

So, in short, if you will, the singular event must be – perhaps we have to say it this way; sometimes mathematicians say it – *isolated*, that is, at heart, being different from its neighborhood, and perhaps even it would have to be isolated in space, that is, having no neighborhood, no one in the neighborhood. It's interesting that you spoke about Borges earlier, etc., because I can cite a perspective of this sort. At bottom, [Henri] Poincaré said that a singularity is a deformation; this is Poincaré's point of view, and this can be shown rather simply in the example – then, I would like, nonetheless, excuse me, since Leblanc, Maurice Leblanc… --- This passage is a bit brutal, -- [*He goes to the board*] but I would like to remind you precisely why an inflection is a singularity.

Inflections – I am not drawing, but… -- An inflection is what would be like this. [*Pause, while he moves toward the board*] Here's the inflection, it's there. Why is this a bifurcation without seeming to be? Because, I want to say, if everything went well, the curve should go like this, that is, it should have behaved here like this, that is, being in a tangent and climbing to the complement, and there, it should have continued like that, without this direction entirely. It's not an angular bifurcation; a bifurcation is not necessarily angular, but as soon as I draw this part, we understand why the inflection is a singularity because, at bottom, suddenly, something happened; the function has moved into another direction. [*He sits back down*] There's an example of the notion of bifurcation, that is, Poincaré, at that moment, when the discussion unfolds, that is, at the end of the nineteenth, beginning of the twentieth century, he proposes this idea that the singularity is a bifurcation.

So, for singularity to be defined – and here I return to the neighborhood – for singularity to be defined, all that's around – and here, we come back [*unclear words*] – all that's around would have to be regular, that is, as close as I might be in the area – and there, we see Leibniz return – as close as I might be in the area around this point, that is, in the neighborhood – I am avoiding the term "neighborhood" because it has a very precise mathematical sense; this is why I am using the rather poorly chosen term "in the area", which isn't very clear, because the term "neighborhood" does not have a similar sense; in the end, it has a precise definition. So, all around there must be a certain regularity. And this is perhaps anecdotic, but it nonetheless is perhaps fundamental: one of the ideas that bothered mathematicians a lot at the end of the nineteenth century, is: could there be a system in which there are only singularities, and how would we recognize such a system? That is, can we describe a situation in which, at bottom, all the points are singular, and how would they be singular definitively if they are themselves alongside a thing that is itself singular? Logically, my definition doesn't work well because singularity, meaning having a different behavior from what's happening all around, provided that the behavior of what's happening all around is itself regular. If it's not, we are in a strange singularity.

So one of the examples, one of the greatest creators of strange objects in this period, has to be [Georg] Cantor, to whom we can render… He is the one who produced a singular object, around singulars, and at the same time maintaining certain regularities. I am describing for you what is called Cantor's discontinuum; it's a strange aggregate, but that's quite interesting. The system is very simple, so I can describe it to you. Now, as for analyzing it more closely, unfortunately it's a bit difficult technically. But by thinking about this, you can perhaps imagine some aspects.

The technique is simple. Cantor takes a segment, [*He returns to the board*] he divides it into three, and he takes out the central part. It remains two pieces, dividing it into three, dividing it into three and taking out the central part. That's continued indefinitely. Each time, the central part is removed. In the holes, the space, one does surgery… as Deleuze said, and at that point, one obtains a set that, at the limit of this infinite production of singularities, all the edges become singular points, everything that is on the edge of a hole. At the end of this technique, what remains is this when one follows this procedure indefinitely. It's that all the points are singular, and they are at the edge of a regularity because each one is located at the edge of something, the points that remain; in some ways, they are all potentially singular. That is, we obtain a kind of situation of singular points at the edge of regular points, and in fact, they are all singular because at every instant they will become so at each instant along the edge of something. The complete pieces of this system are more complex. We're forced to pursue a small technical presentation, so if you will excuse me, I couldn't do so because… [*He doesn't complete the sentence*]

Cantor's discontinuum leaves one perplexed (*fait rêver*). There followed another entirely astonishing example of regularities and singularities that is quite strange, and that cannot be completely described either. It's a curve known as the Peano curve, but there are two or three examples. The Peano curve is a kind of… it's an entirely classic curve; it's continuous. It's drawn inside a square, but if you draw it all the way to the end, it entirely fills the square. That is, in some ways, if you try to draw it, if you try to trace it point by point, one obtains the following thing, at bottom, some kinds of arabesques which, if pushed all the way to the end, fill the whole square.

Deleuze: But that's the same thing as Mandelbrot.

Maarek: Yes, Mandelbrot made use of that. At the moment when the Peano curve was published, around the year 1900, at that moment, this is the central problem of the singularity in the world of singularities.

Isabelle Stengers: [*A barely audible comment, regarding the introduction of a text by Jean Perrin*, The Atoms, *that Mandelbrot will quote*][11]

Deleuze: He even says – so we're going to be finishing for each other, [*Laughter*] which is great, it's so Leibnizian what you are saying that it's astonishing. That gives Leibniz such a presence in modern mathematics – that before the Perrin text that Isabelle mentioned, the words are "infinitely cavernous" or "spongy". He says that matter is not at all continuous, the new vocabulary of discontinuity. We can always create holes, and it's the idea of an infinitely cavernous form, it's the same thing as the curve of "corles", infinitely cavernous, infinitely spongy. So he takes "corles" as an example, like charcoal. In fact, for those who are interested in this, this is in a book by Mandelbrot, published by Flammarion, called *L'objet fractal* [*The Fractal Object*]. And Mandelbrot has a long quote from a text by Perrin, who is a great physicist and starts off from this quotation.[12]

Maarek: I'd like to go back a bit. That is, let's leave aside momentarily these objects, these singular points that would themselves be immediate neighbors of points [that are] themselves singular. Let's place ourselves momentarily in the good situation that is a bit like the one you were describing in the last class. That is, the singular point is surrounded by regular points; if all goes well, an event is isolated – the word "isolated" in fact belongs to math vocabulary, but its interpretation is entirely simple, as able to be situated precisely.

However, we still have a problem in this case that we have to consider. It's this: how are the neighborhoods of a singular point constructed? So this refers us into the heart of the notion of neighborhood, of what happens immediately after, that is, to the heart – and here, we must refer to the Leibniz of differential calculus, that is, to the notion of the infinite – because you inevitably notice as well that in the past classes Deleuze as well as me right now in the examples I am going to provide, we always necessarily use a measured procedure, such as Deleuze the last time alluding to convergent and divergent series going from one singularity to the other – the notion of series refers us to the infinite in a certain sense. That is, it's not a question of making [*inaudible*], but of speaking about what is infinite in this area (*alentour*). Later we will try to give a definition of what Deleuze has used for quite some time, in any case, he used it in his own way, the notion of convergent series and divergent series, around or toward or between singularities. For the moment, we are staying with the area (*alentour*).

So, the principal adventure is what to do, how, and why, and under what form do we need the infinite? So if you allow me, I am going… This is how I have a little… I think that this will be clearer and easier. I am going to move us very quickly through time and reach 1934 and then return to Leibniz afterwards, because the adventure that is to unfold begins in 1934 and will end with Leibniz like a flashback.

At the start of the… In the first decade of the twentieth century, there was a mathematician – this is an adventure that everyone know, but I am going to recount it very quickly so that there won't be any problems – there was this kind of axiomatizing fever – for a pile of reasons that are historical and that are based on the crisis of geometry and on a series of events like that – there was a terrible axiomatizing fever. That is, people wanted to axiomatize theories. Axiomatizing meant giving the fundamental, indispensable rules allowing one theory or another to be built… [*Interruption of the recording*] [1:22:51]

Part 3

… that is, whole numbers. Of course, let's be clear, people knew for a long time how to count, that is, in a daily basis, they didn't axiomatize a theory – in the end, we already have… Look closely at the situation as it unfolds: there is a universe that is, for example, a universe of numbers. They already know how to use them, they've known how to count for twenty-five centuries -- and Bourbaki claimed there were even mor -- so they knew counting, they knew lots of things in arithmetic, and in 1910, they axiomatize. The axiomatization of geometry dates from 1890 when they already were able to draw and teach courses of geometry. Thus, axiomatizing is giving rules to the set allowing defining to happen in an explicit way what one needs. We'll see later an entirely surprising theory from Leibniz.

So then, there is this long, long work of axiomatization. It's a complex problem – in the end, everyone who knows the history of math knows about all this work that unfolds very, very precisely and that arrive at 1930 or 1932 with a book by [David] Hilbert, that is *Grundlagen der Mathematik* [1928] which is the great axiomatization of the set, the same edifice at that moment as the one… it was indisputable. At that time, a mathematician named [Thoralf] Skolem posited an astounding problem; he says… he sets forth the inverse problem. If you will, I will recount this as an anecdote.

Imagine that you have axiomatized a theory and that a Martian arrives, who knows nothing of this theory, nor about axiomatization – we are assuming it's a Martian or any other extraterrestrial knowing how to read. So he's going to follow what you say within the axiomatic; he's going to apply it and regularize it. He's going to apply them (*rules*), entirely regularly. He knows how to read, and he knows how that functions, and suddenly he is going to engender, he must engender the model that you've axiomatized from the start, that is, he must uncover – perhaps… let's say, let's do this experience because [Thoralf] Skolem did it. His article is all about this point. Let's imagine that we have the axiomatization of arithmetic, that is, we have all the axioms of arithmetic. Little by little, the extraterrestrial in question, who knows how to read this axiomatization, is going to create some numbers. Perhaps he will give them different names than those you are used to giving them in your language. But after all, this is not upsetting because from one individual to the other on earth, we already give them different names. But counting in English, Arabic, or in Chinese, or in French, never caused problems; the two sets of numbers, belonging to the Chinese, Arabs, etc. are isomorphic, as mathematicians say. They have the same forms, and they have the same composers. So, they differ only by their names, and this is what happens in our universe.

But Skolem posits the following problem: doesn't he [the extraterrestrial] risk finding another universe of numbers, one that is completely different and that obeys the same axiomatic? And so, you imagine the strange result of this order; this is effectively what Skolem shows in this 1934 article I mentioned. He shows that he can find many other systems of numbers that are not at all similar to the preceding one, to those with which they were familiar. He pushes this undertaking further by showing that we cannot axiomatize a set like that of numbers in such a way that the model obtained might always be the one that we thought of previously. Logicians use an expression; they say that a system of axioms that would only produce a single model is a categorical system of axioms, and a system of axioms that would not always produce the same model is [*unclear word*], so Skolem demonstrates in this article that it's not possible to create a categorical axiomatization.

And here we are going to see the problem of finite and infinite appear in a strict way because we are thinking of, and at bottom, what Skolem describes is the following situation: if the model from which you started in order to axiomatize was itself infinite, there is no categorical axiomatization. There is categorical axiomatization only if the starting model is a finite model, and at that moment, the axiomatization is not at all a learned operation. It comes down to naming persons, that is, it says, I would only receive this person, Mr. So, and Mr. Such, and Mr. So, and at that point, it's not an axiomatization, but rather an entirely simple restrictive rule. And most of the time, the world of which we're speaking and the world of singularities and neighborhoods, is a necessarily infinite world, thus not axiomatizable, in any case, not axiomatizable in a categorical way.

So, where is the problem? You are going to see why Leibniz is going to come back into this story. Skolem says this definitively: it's that in the model that my extraterrestrial will have produced, what happens, and that I cannot prevent him from doing in the axiomatic? It's that outside of numbers – call them (in French), zero, un, deux, trois, quatre, cinq, etc., that you know – he can introduce some new ones, others, that obey the same rules as ordinary arithmetic. Of course – here I am situating this precisely because you will yet see that it's important; excuse me for making this a bit technical – of course, there is one of the Peano axioms that says that zero is a number that has no predecessors, so it's the first among numbers. Then, obviously, since there's an axiom that says that, my extraterrestrial cannot create a new first number. He is entirely forced to take the same one as us. Only, the new numbers, if he wants to, he can place them after all the others – not after zero since there will be 1, not after 1 since there will be [*inaudible*] than us, but he can place one of them quite far, and nothing in the axiomatic forbids him doing this and nothing can state that he is forbidden from introducing a new number because imagine what would happen: in order to introduce a new number, you would have to know an axiom saying, "I forbid having any number entered from outside the ones that I stating and these are the following." And how would you state them since there is an infinity? You are incapable of stating them.

You therefore see why: if the number is infinite, you cannot make axioms that forbid numbers, foreign ones. And what is going to happen is this: imagine how my extraterrestrial, by placing one more of them, this 1 is going to have successors, and is going to have predecessors; nothing forbids it, to the contrary. The axiomatic allows him to do this, and he is going to proliferate [numbers], and there will be 1 double, 1 triple, and so on. Why try to prevent him doing so? It's going to proliferate. Consequently, at that point we will see numbers appear that are larger than all the numbers that we already know, and these numbers that are larger than the ones we already know, these will already by infinite numbers.

But careful: it's not at all a question – for those thinking there might be a misunderstanding – it's not at all a question of Cantor's infinites; it's an entirely different problematic. It's a question of numbers that have the same arithmetic properties as everyone. That means that it's where we are, we stay in the universe of arithmetic, and there will be evens, there will be odds, after an even, there's an odd, after an odd, there's an even, etc. etc. So here we are in a universe in which there are new numbers that are larger than all others – let's call them infinite in quotes – but that are whole numbers like everyone. Later we will see – I will tell you about the text because it interests me greatly – you will see that the first one who had the idea was Leibniz, textually, [*several inaudible words*].

So we will be in an entirely proliferating universe, but one that we cannot forbid, that can be very vast, or quite reduced. That will depend on my extraterrestrial and his desire, etc. So there we have something that we can construct which is – logicians call it this way, but the notion is quite simple to understand – an extension of the set of whole numbers (*entiers*); [*Sounds of cassettes being changed in several recorders*]; so it's a vaster set in which obviously one can discover [*unclear word*] of whole numbers, and this extension itself is going to include a certain number of numbers, in the most banal sense of the term, but that have the property of being infinite like [*inaudible*].

So there we have the first article, the one by Skolem. From that point onward and into the 1960s, an American mathematician by the name of Abraham Robinson, who died in 1974, tells himself: couldn't we do the same thing, not only on the wholes numbers (*entiers*) because the properties described by Skolem never cease being equally applied in real numbers, that is, in numbers that you know, that is, the total set. In mathematics, we call these real numbers, that is, the set of all there are, decimal numbers, rational numbers, irrationals, etc., the complete set of numbers, what are usually called the number set (*l'ensemble de nombres*). The work is relatively delicate because the theory of real numbers is an axiomatic a bit more complicated, as you might suspect, than the axiomatic of whole numbers, -- and we'll be coming back to this later -- but Robinson does the work and shows that it's possible. At that point, we are going to obtain a set of numbers in which there will be infinite numbers as in arithmetic; there will be their inverses that will be infinitely small insofar as they are numbers this time. That is, these are not infinitely smalls as I was using the term earlier for the series. These are infinitely small insofar as they are… they become very small; these are not infinitely small limits, as mathematics is used to designate them. These are veritable numbers that are infinitely small perhaps quite simply because these are the "un-sure" inverses, an infinitely large, an infinite in quotes. So if you take an infinite in quotes, you are going to have, by taking its inverse, a number that will be smaller than all others.

Faced with this difficulty, if you will, of this extension, a name was needed for these old numbers. So Skolem found the name “standard”. We call standard the usual numbers, and we call the others “non-standard”. “Non-standard” has such a vague interpretation in English that perhaps you’ll allow me this [gap]. So this theory will be called – so it’s developed around the 1960s – this theory will be called “non-standard analysis.”

I’d like to recall a point that I said in passing earlier, but that’s entirely fundamental. It’s that this new universe with the infinitely large and the infinitely small strictly respects all the axioms and all the rules of the traditional real (number) set. That is, every theorem that has been demonstrated in the set of real [numbers] is a theorem in this new universe. We push farther by showing – but this is a tiny bit more complicated a problem – that this extension is conservative. That is, everything that can be shown in this new universe is equally true in the limited universe. Currently, mathematicians are considering a special little case, a theorem that Robinson was to prove about this extended universe in the 1970s. We ought to be able to find a proof in this limited universe. It’s a theorem – here I give a small anecdote – that up to now has not been found, which is somewhat embarrassing. In the end, theoretically, that should be possible. This theorem known as Bernstein-Robinson for the moment has no equivalent, at least so far, no equivalent yet in the real universe.

So, if you will, now I am going to come back to Leibniz. Here we are now facing a new universe in which, without any new incoherence, we have a language in which there are infinites, infinitely smalls that behave like numbers and have exactly the same properties as them without any differences, that is, we are going to be able to define and say what needs to be said in these universes that are vast universes. [*Pause*] And it’s here that Robinson realizes that in the end, this is Leibniz’s idea, one that he had. In the final chapter of his book – because I believe that this construction is written in a book published in 1966 in Amsterdam called *Nonstandard Analysis* – in the final chapter of his book, chapter 10, Robinson comes to cite the reference of origin, and at bottom, he cites Leibniz’s theory of infinitesimals. Because this might be of interest, I point out that a short while ago at Blanchard press, there appeared the entire collection of Leibniz’s works on the calculus of infinitesimals. So I am going to be returning to Leibniz shortly, and will do so almost right now. But Robinson cites a letter, from 1701, no, a report (*mémoire*) by Leibniz dealing with his feelings about differential calculus. It’s a report that he wrote in 1701, that is, rather late in his life, and later I will tell you why I’m insisting on that fact that a report written late. And Leibniz writes, the text is in French; he’s cited in French in the book:

"There is no need to take the infinite here rigorously, but only as when we say in optics that the sun rays come from a point infinitely distant, and thus are regarded as parallel. And when there are more degrees of infinity, or infinitely small, it is as the sphere of the earth is regarded as a point in respect to the distance [of the sphere] of the fixed stars, and a ball which we hold in the hand is also a point in comparison with the semi-diameter of the sphere of the earth. And then the distance to the fixed stars is infinitely infinite or an infinity of the infinite in relation to the diameter of the ball. For in place of the infinite or the infinitely small, we can take quantities as great and as small as necessary in order that the error will be less than any given error. In this way, we only differ from the style of Archimedes in the expressions which are more direct in our Method and better adapted to the art of inventing.”[13] [*Pause*]

A little bit farther on, in another text, no, in the same one: "And it happens that the rules of the finite succeed in the infinite as if there were atoms, although there are none at all, matter actually being subdivided without end; and that vice versa, the rules of the infinite succeed in the finite as if there were infinitely small metaphysics, although there need be none; and that the division of matter never achieves infinitely small parcels: this is because everything is governed by reason and, were it otherwise, there would be no science nor rules, which would not at all conform with the nature of the sovereign principle." Same text by Leibniz.[14]

Finally, I will quote for you a final statement from Leibniz, as I will come back to this passage. It's a letter from Leibniz to Varignon, in 1702: "Between you and me, I believe that Monsieur de Fontenelle, who is of a courteous and beautiful spirit, wanted to make fun of us when he said that he wanted to make elements of metaphysics out of our calculus. To speak frankly, I am not so persuaded myself that one must consider our infinites and infinitely small other than as ideal things and as well-founded fictions. [While] I believe that there is no creature beneath which there would be an infinity of creatures, I don't at all believe that there would be, nor even that there could be any infinitely small, and this is what I believe myself able to prove."[15]

Notice that Leibniz is uncomfortable in this text. This is why I insist on the fact that these texts come from the end of Leibniz's life. In Leibniz's first texts on differential calculus, at heart Leibniz thinks that it is possible for there to be some infinitely smalls in a strict sense. You know that differential calculus had been invented almost simultaneously by Newton and Leibniz – I won't insist on this, but it was perhaps Newton who did so a bit earlier -- but the two presentations are, let us say, they entered however into a long dispute that didn't unfold very elegantly. Outside the problem of primacy [who invented first] which is secondary, what is interesting to note from a strict viewpoint is that when he presents differential calculus, Leibniz would like… he seems to say, his vocabulary is different, that there are infinitely small objects, infinitely small meaning smaller than any assignable finitude – it's in this way that he defines it – and these infinitely small objects are useable because they function as functional numbers. These are some infinitely small that can be used clearly. Earlier I wanted to present another quote from the same report. Newton has a more complex presentation because he uses alternatively sometimes a language in which he suggested there are some infinitely small, sometimes a language in which he says that this is a limit, or sometimes a language that is closer to the classical description.

In the universe of mathematicians, in 1790 as in 1987, the reaction is the same, that is, introducing new numbers is very disturbing, and even unacceptable, and mathematicians pretend they might be somewhat sympathetic, and yet even with Robinson's work, they feel uncomfortable. As an example, I will cite a talk by an entirely well-established mathematician called [Jean] Dieudonné, in a talk from 1980 or 1982 more or less, in which he says that introducing new numbers results in making mathematics empty and insignificant. And Dieudonné was a founder of the Bourbaki group. In passing, I point out that another Bourbaki member, Claude Chevalley – a former professor in this very university – wrote in a text saying that, at heart, doing a kind mathematics, a bit like [René] Thom, doing an absolutely rigorous kind of mathematics is to make it insignificant. I was able to ask Chevalley about his ideal viewpoint, the opposite of the text I was discussing earlier.

The central question… At that time, Leibniz encounters the same difficulty; that is, to say that there could be some infinitely small as object and they would obey the same modes of calculus, the same approaches, as the other numbers appeared quite unacceptable. And thus there immediately was a polemic. This is the allusion he made to Fontenelle; this is even an allusion to the Marquis de l'Hôpital shortly thereafter who had been rather won over by the infinitely small.[16] So, in this 1702 text, Leibniz says, I am returning to this; it's a polemical position, one that is a retort, "[one must say] I am not so persuaded myself that one must consider our infinites and infinitely small other than as ideal things and as well founded fictions." But outside of his doubts, Leibniz continues to defend his idea of infinitely small because it's the only way to speak about neighborhood. And here, I return to the [*inaudible*] comment.

What has Robinson proven? Quite simply that this idea is not logically untenable, that there is no possible contradiction between it and the current idea of real-ism. At bottom, he simply did this – and I believe this to be the essential point of this work – it's to have shown – a series of projects about which I am speaking– that we can have a language including these objects, these "well founded fictions" as Leibniz says, and that causes absolutely no contradiction. That version of infinitesimal calculus will never gain prominence. In his book *Nonstandard Analysis*, in its central chapters, Robinson presents a veritable course on mathematics, that is, he describes what would be the description of a course on ordinary analysis in the other universe. There is a point that is entirely appealing in this text; it's that all notions become perfectly simple thanks to the infinitely small. For example, if some among you have ever had to suffer through the definition of a derivative, read it in Robinson, and you will understand it completely. You will wonder how it was that you didn't understand it before. This is a notion that becomes immediately simple in this [other] universe because instead of having – you remember the derivative perhaps in secondary school. […][17] This definition will take centuries to arrive. Creating a relatively precise definition in terms of limits will take centuries to arrive. And since Deleuze mentioned him the last time, I point out that the edifice is crowned by Weierstrass, a mathematician who you mentioned in the last session, who creates a very complex definition of pure angles.

If we have the infinitely small, then the definition of the derivative, of the limit, of a convergent series, all that is quite simple. What would happen then? Quite simply this: every point is found to be surrounded by a zone of infinitely small situated infinitely close to it, in the sense of infinitely small. That is, it will be situated with, all around it, a small halo, a small cloud, and this cloud surrounding it is formed by those that are infinitely close to it. To this set "point plus its surrounding little cloud", Robinson will give the name "monad". And at bottom – and reading Robinson's text shows this very well, and of everything done afterward because there were works obviously based on *Nonstandard Analysis*… -- Reading this text show this: that the relations of a point and what surrounds it is a relation of this monad with the universe that surrounds it. I will later give an example that, in my view, goes in the direction of what Deleuze was saying. [*Pause*]

The monad therefore is a universe formed of points and of that which is infinitely small for it. This requires logically that we undertake a somewhat complex little task of defining equality. We can no longer have in a theory of this kind, in a universe of this kind, a sole notion of equality, as there was in the classical universe. That is, we could have two definitions of equality when it's strictly the same object or when these are two objects that differ by an infinitely small.

Notice that this example is not prohibited from classical mathematics, right? I would point out in passing that without any risk, we can write – I will write this out; it will be easier for you to see – A = 0.99999 … because there is no difference between 0.9999 – if you take my elliptical points out to the end – and 1; the two numbers do not differ at all. These are two ways to write out the same number. One is presented as an infinite approach, that is, if you are lazy and you stop writing the nines at some point, the equality is false. But if you follow this through to the end, it's the same number. Thus, even in the matter of equality, we have a problem. So obviously, in the Leibnizian universe of the infinitely small, one has to distinguish between the two aspects. In that universe – I'd like to answer your question – to say in passing that Leibniz gives an example of the same thing that I do, in the same text cited earlier from 1702.

I would now like to describe in that case what the notion of convergent series can be. So I am choosing only one example in this description, that is, I am rather quickly going to draw a parallel between the notion of convergent series in the classical mathematical universe and the notion of convergent series in the universe of the infinitely small. A convergent series is what? A series of triangles tending – or rather, Deleuze often uses the term "convergent series", even in *Logic of Sense* long ago; I think that for mathematicians, it would be more correct to say "convergent succession," (*suite convergente*) but in that case, it's not at all serious, not at all serious. Let us call it "series" if you will.

This is a series the terms of which are going to come close to a given point; that is, we will have a point [*He goes to the board*] – that I draw like this – that will probably be outside the series, but the series will come close to it. What does "come close to" mean? Here I am speaking in classical terms. In classical terms, "to come close to" means that each time that I situate a circle near the series, there is an intimacy of terms in the circle and finite number of terms that are outside it. If I take a still smaller circle, there will be more of them there [*He points to the drawing*] than for the rest, that is, there will be an infinity. That's what the idea of converging is, that is, if you will, coming close indefinitely, or as the mathematical treatises said at the start of the [twentieth] century, being as close to the limit as one would like. When we read these texts, they always perplexed mathematicians: who is this "one", what does it "want"? Anyway, this is parenthetical. But in the end, within the doxa of the old treatises, it was written that one goes as close to the limit as one would like. That means, as close to the limit as the circle that I indicated. This is how Weierstrass sums up the problem by making smaller and smaller circles, by showing that the series is convergent if there are an infinite number of terms in the circle and a finite number of terms outside. This is the rather complex definition, you see, because it requires that we create circles. The solution, there's a choice; I draw it for you here: it states that each time that you will provide a circle, I would find an infinite number of points in the circle if it's convergent, and a finite number outside the circle. So, the solution that Weierstrass proposes is [*He cites a formula*], that can be found.

So let's now place ourselves in the Leibnizian universe of the infinitely small. The convergent series is one that crashes, that enters at one moment into Leibniz's monad, that is, whose difference of terms with the limits will become infinitely small at a certain moment and at the end of a finite number of steps, and thus it enters at that moment into the monad. This doesn't mean that it is then… If you will, at that moment, I have a tool to describe this hole, it's the monad, this kind of reflection of the universe around this point. That is, at bottom, there is going to be – and here we see the relation between finite and infinitely small, between infinity and infinitely small – at bottom, the monad is a kind of reflection of the outside universe, at the interior of the area (*alentour*) around the point, and if this is a singularity, around the center. Robinson demonstrates in fact – it's quite astonishing, he was answering the question that Deleuze asked at the last class. He shows that there is only one case in which this procedure would not work: it's if the point in question is isolated, because if it is isolated, I mean if there is no one around it, the monad would be reduced to itself, and this is an entirely special case.

Deleuze: Leibniz said this as well, precisely.

Maarek: Leibniz said it as well. It's a theorem, if you will, in Robinson's book. So, if you will, here is what that universe would yield. This description is rather odd because in reading the books, if one were doing a math course with *Non-Standard Analysis*, these notions would seems really simple [*several unclear words*] because we have a set of words in able to speak of the limit. And this idea that is found in Leibniz's 1701 report, that is, when he says it's an ideal set, it's a "well founded fiction," I would say that I almost want to say, if we could, that he didn't show that it was well founded, but the proof would take a lot of time. In any case, an idea is located there. This whole 1701 report, which is called – it's in the book published with Blanchard – "A report by Monsieur Leibniz stating his feelings on differential calculus" is there in the Guérard edition, page 350.

Deleuze: In the mathematical [works]?

Maarek: Yes, and there one finds a French translation. In fact, it's written in French. We would have to read in depth the whole of this construction.

So, to close, I want to mention something: that in history, before Leibniz, there was only one mathematician who came close to this idea. This mathematician – it's not at random since Leibniz cites him in his text – is Archimedes. And Archimedes, with extreme care as well, came to this idea, but I almost want to say, he did so in a negative way. That is, at the moment he came toward it, he rejects it, quite clearly. So I want to close by speaking about Archimedes. Where can we situate this rejection? In one of Archimedes's texts concerning the calculation of lengths, of volume, and of surface. He found himself faced with this situation: if you take a small line and you add a point, do you increase the line's length? Let's take that differently: if you were given some points, could you make a line with them? The answer is no; in any case, Archimedes's answer is no. That is, the infinitely small cannot create a line of infinitely small points. Just as in stacking up lines one beside the other, you cannot obtain surfaces; something will always be missing. There will be holes all the time; there will be singularities all the time. For example if you were given some segments and you had to make a square, a triangle, I don't know, by stacking them… Just as with surfaces, you couldn't create volumes. In saying this, I am almost quoting Archimedes textually because he takes up the three examples one after another. Consequently, he collides with a difficulty that is: what does one do to stack up points to create lines? And he decides to adopt the position to reject this, that it cannot be done. With points, one cannot obtain lines, Archimedes says textually; with lines, one doesn't get surfaces, and with surfaces, one doesn't get volumes. He insists on this, repeating it several times.

What would we do then with the infinitely small, well, provided that we also have infinites, if we have infinites, that is, if I can grasp an infinity? So the fiction is located at both ends in order for that to function. I have to be able to grasp an infinity of infinitely smalls in order to locate a classically standard object, [*several unclear words*]. And all through the text, Leibniz indeed senses this difficulty because each time there is an imbalance of infinitely smalls. And I believe that one of the ideas that would be, probably, acceptable (*tenable*) and attractive is this: what Robinson calls the monad around a point – and this will be interesting to see at the level of the singularity -- it seems to me personally to be acceptable for this idea to govern the relations between the singular point and everything that's around it. The example of the convergent series is entirely indicative. That this would be a divergent series – I am going to close with this – that would be a series that doesn't enter into the monad, the monad in Robinson's sense. So there we are. I can answer questions… I don't know if what I said is very lateral.

Deleuze: Oh, not at all, not at all. So this, I find this magnificent. I find it magnificent for us because of all that it brings to us. You understand, I am picking up on one of the topics he discussed: here we have a modern mathematician who is led to use mathematically the notion of the monad. So that enlightens me as well because, well, he belongs to your department, Gilles Châtelet never stops his mathematical work, in which he needs the notion of monad, that is, a disciple of Robinson. And what interests me is the mathematical definition of the monad in Leibniz. And he tells us, if I followed correctly, the monad is formed by singular points with what happens around, once it's said – with what you called the "cloud" around – once it's said that the "cloud" and "around" are strictly defined mathematically. So if I'm told and if I learn that it's a mathematical notion and that there is an actual value there, notice immediately what I have there: I say, such mathematical monads are precisely individuals because the definition – I don't know if you are feeling this – coincides completely with what I wanted to say: a condensing of singular, prolongable points, that is, with [*unclear word*] around, defined by the prolongation in all directions, this is exactly Robinson's definition. I didn't know about it, but this a pure joy.[18]

So I would almost say, no, for me, this is a presentation with extraordinary richness that you have generously offered thanks to all that you have taught us. I would just like to come back to one point, the only point where I would have problems after having listened closely. It's the point on what you were saying about Leibniz's texts around 1700 because – I'm opening a parenthesis before moving quickly to this point – you show very well that in Skolem, infinite numbers submit exactly to the same rules, that is, the axiomatic. So you say, especially one must not confuse them with the transfinites, if I understand this well.

Maarek: That's right.

Deleuze: One must especially not confuse them with Cantor's transfinites, because the transfinites don't belong to the same axiomatic as… That's it, right?

Maarek: Yes.

Deleuze: So the numbers, didn't he have a special name to prevent confusion? He says infinite numbers.

Maarek: He says nonstandard infinite numbers.

Deleuze: Ah yes, there's nonstandard. I didn't know about that. How do you spell Skolem. [Maarek *spells it out for Deleuze*] What years was he writing?

Maarek: His writings are especially from around 1924. As an aside, the same year I mentioned, one I didn't discuss, he's an author of a paradox, known as Skolem's paradox …

Deleuze: Ah, I don't know about that.

Maarek: … that is really astonishing because one of the ideas located in Skolem's article, it's to tell himself – you remember earlier I said that an extraterrestrial could have learned arithmetic; so obviously, the mathematical model that we have in our head – 0, 1, 2, 3, 4, 5 – is smaller because what I've said, the extraterrestrial can extend this system. So one of the responses that have been given by Skolem is: yes, fine, but the model we have in our head is always the smallest, and there where the system is paradoxical – and this is what's called Skolem's paradox – is that if an extraterrestrial were given the axiomatic of set theory, the model he would create would not be the smallest. The model he would create can be smaller than the one we have in our heads. That is, there is the smaller model than the one we know, [Deleuze: Yes, yes, yes] that is, in the end, there are models, I would go even farther, there are non-countable models that contain non-countable sets, it's what we currently call the minimal model. So this is paradoxical, it's paradoxical; the solution to Skolem's paradox is not simple. In passing, I will suggest that in the book *La théorie axiomatique des ensembles*, Jean-Louis Krivine – who is writing, as it happens, a very, very beautiful book[19] – resolves Skolem's paradox with a pirouette. It's yet another paradox.

Deleuze: What language does Skolem write in?

Maarek: Skolem writes in German. The 1934 article is in German, but he is Scandinavian [Norwegian].

Deleuze: So concerning the texts from around 1700, I'd like to present things like this. The texts you referred to, Leibniz constantly says, or seems to say: you know, one must not exaggerate, especially against Fontenelle. The question is not of knowing if the infinitely small exist or not. The question is by what mode of calculation the infinitesimals work, function. [*End of the recording; Deleuze may well continue his response*] [2:09:18]

Notes

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[1] The following text, up to minute 59, is the transcription available on WebDeleuze, supplemented with several additions from the BNF recording; the subsequent 70 minutes are newly transcribed and translated, drawn entirely from the BNF recording.
[2] Cf. the start of chapter 5, *The Fold* (University of Minnesota Press, 1993), p. 60; *Le Pli* (Minuit, 1988) p. 80.
[3] See Cinema session 3 (29 November 1983) for the development of these three texts, by Leibniz, Borges, and Maurice Leblanc.

[4] From *Theodicy* online, para. 405, http:// http://www.gutenberg.org/cache/epub/17147/pg17147.txt (accessed 17 March 2024).
[5] This is no doubt a reference to Heinrich Wölfflin, *Renaissance and Baroque* (Ithaca, NY: Cornell University Press, 1987; Paris: Livre de Poche, 1987), mentioned in session 7, 20 January 1987.
[6] Cf. *The Fold*, pp. 62-63; *Le Pli*, pp. 83-84.
[7] Jorge Luis Borges, *Ficciones* (Grove Press, 1962), p. 98.
[8] The narrative of this novel by Leblanc is also in *The Fold*, pp. 62-63; *Le Pli*, pp. 83-84.
[9] The WebDeleuze transcript ends here; the following text (70 minutes) is transcribed from the BNF/YouTube recordings.
[10] Deleuze's mathematician colleague, Marcel Maarek, will be named (with his last name) by Deleuze in the following class, on February 3, when Maarek is absent and Deleuze refers to the presentation he makes here. An oddity is that even in the summary of this class session edited by Frédéric Astier (*Les Cours enregistrés de Gilles Deleuze, 1979-1987* [Sils Maria, 2006]), Astier only designates this invited colleague as "an intervener (mathematics)" (p. 167). Marcel Maarek joined Vincennes the year before Deleuze, in 1969-70, and among other courses, he taught logic both in his own department as well as for the philosophy department. See Octaviana Bibliothèque numérique for archives on Vincennes, https://octaviana.fr/items/browse?collection=633.
[11] Jean Perrin, *Les Atomes* (1913; CNRS Editions, 2014) (*Atoms*, 1918).
[12] Deleuze referred to Mandelbrot on this same topic during session 2, 4 November 1986, as well as in session 3 of the Painting seminar, 18 April 1981. See Benoît Mandelbrot, *Les Objets fractals* (Paris: Flammarion, 1975; reed. coll. Champs, 2010), *Fractals: Form, Chance and Dimension* (Brattleboro VT: Echo Point Books and Media, 1977; reprint ed., 2020).
[13] Cited in French in Robinson, *Nonstandard Analysis* (Amsterdam: North-Holland, 1966; third edition, Princeton University Press, 1996), pp. 261-262.
[14] Cited in French in Robinson, *Nonstandard Analysis*, p. 262.
[15] Cited in French in Robinson, *Nonstandard Analysis*, pp. 262-263.
[16] See the quote in *Nonstandard Analysis*, p. 263.
[17] He quotes several formulae and tells his own experience as secondary school professor and the typical experience with derivatives of students at that level.
[18] Deleuze refers to Robinson's *Nonstandard Analysis* in *The Fold*, pp. 129-130; *Le Pli*, p. 177.
[19] Jean-Louis Krivine, *Introduction to Axiomatic Set Theory* (Dordrecht, Netherlands: Springer, 1971).